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# Seismic shear response of slab with distributed mass for linear-elastic bay

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**Abstract.** Common concrete buildings have concrete floor/roof slabs without lateral braces, as well as steel deck slabs with steel frame. In routine seismic design of the slab for in-plane shear, the distributed mass on the slab would be usually treated as the physically lumped structural mass on intersections of the slabs with bays. Therefore the obtained slab shear entirely depends on a difference of the bays' displacements and the slab stiffness. Meanwhile as for the structural system including mass distributed on the slab, the seismic behavior has hardly been clarified. This study addresses the maximum local in-plane shear response of the slab with distributed mass of single-story linear-elastic systems and its formulation. Although this study focuses on such elemental structural system, the results and conclusions could be useful for fundamental consideration of multi-story elasto-plastic ones. For the purposes, a variety of single-story systems are investigated using linear-elastic time history analysis under earthquake. Then two predictable formulae for linear-elastic system including bay and slab are newly proposed. Moreover the proposed predictable formulae are verified with the analytical results as well as previous ones for lumped mass system. The comparisons show the validity of the newly proposed formulae in this study.

*Keywords: Seismic response; Slab; Diaphragm; Shear; Mass*

## 1 INTRODUCTION

### 1.1 Objectives

In common, floor/roof slabs usually might be designed for in-plane shear due to seismicity using a difference of facing bays' displacements and the slab's in-plane shear stiffness. That is to say, a shear deformation of the slab in design procedures might be expressed by supposed lateral brace members instead of the slab diaphragm. Hence no influence of mass distributed on the slab might be considered. Consequently, such usual design procedures suppose uniform in-plane shear in the slab in each direction. That is on the premise that mass distribution on the slab might cause no shear response in addition to the physically lumped mass system. Does not structural design of the slab for in-plane shear have to consider mass distribution on the slab? To date, no research for this issue has been found to be reported yet. Previous researches have included the following issues. Archer (1963) investigated a technique of formulation of a consistent mass matrix that accounts for the actual distribution of mass throughout the structure in a manner similar to the Rayleigh-Ritz formulation. The natural mode periods and shapes could closely approximate the solution to the exact problem. Goldberg and Herness (1965) formulated the vibration problem of multistory buildings with lumped masses at each intersection between floor and frame, considering both floor and wall deformations. The natural mode periods and shapes could be obtained by use of generalized slope deflection equations. Unemori et al. (1980) studied how the floor slabs stiffness affect the magnitude of the in-plane forces generated in the slabs for multistory building systems with lumped masses. Jain (1984) showed that for long narrow buildings with identical frames and identical floors, the modes that involve in-plane floor deformations are not excited by earthquake ground motion. Additionally it is noted that the result presupposes the acceptability of the lumping of masses of the building at the floor-frame intersections. Nakamura et al. (2006, 2007) have discussed and reported the behaviour and the formulation of dynamic in-plane shear response to the lateral ground shaking for the lumped mass system. However the investigation of seismic shear behavior of the slab with distributed mass had hardly been conducted. Then objectives of this study are:

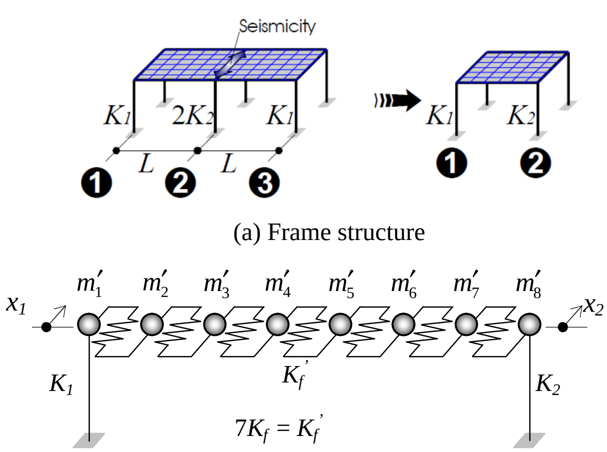
1) to obtain fundamental characteristics concerning local shear response for the distributed mass system of single-story linear-elastic structure as a feasibility study
2) to propose a useful formula for predicting the maximum local shear response in elastic range.

A variety of eigenvalue and time history analyses of single-story structures has been carried out to achieve insight into basic trends in the elastic seismic behaviours of the distributed mass systems. This study may not be directly applicable to multi-story systems. Because it is not evident whether limited study focusing on single-story systems is appropriate for multi-story. Linear-elastic response is comprehensively examined and presented in representative form to provide the framework of the slab design for seismicity. Obtained behaviors and maximum local shear prediction may provide attention to elastic design for moderate to high seismic applications including serviceability limit state. Observations highlighting the dynamics of single-story systems with linear-elastic characteristics might contribute to establish the conceptual framework for envisioning the seismic response of multi-story systems with elasto-plastic characteristics. Through the investigation, a lot of emphasis would be on how the behaviors of distributed mass systems differ from lumped mass systems regarding the slab shear response.

### 1.2 Scope

The scope of this study includes the following:

a. Computation of eigenvalues and eigenvectors of simple linear structural system with distributed mass
b. Computation of dynamic earthquake response of the structural systems with and without the distributed mass by time history analysis
c. Evaluation of results of the eigenvalue and time history analyses
d. Proposal of a formula for prediction of the maximum local shear response
e. Evaluation of the proposed formula with the analytical results as well as previously proposed ones




(b) Analytical model

**Figure 1.** Frame structure and analytical model

**Table 1.** Mass distributions

| $m_1$ | 0.50 | 0.67 | 0.80 |
|---|---|---|---|
| $M_1$ | 0.50 | 0.67 | 0.80 |
| $M_2$ | 0.50 | 0.33 | 0.20 |
| $m'_1$ | 0.125 | 0.4225 | 0.65 |
| $m'_2 \sim m'_8$ | 0.125 | 0.0825 | 0.05 |

## 2 STRUCTRES CONSIDERED AND ASSUMPTIONS

Consider a frame with 1×2 bays such as previous researches (Nakamura et al. 2006, 2007) , according to its symmetry, a simplified model used to analyze in this study consists of two bays with a span *L* (see Figure 1). The structure has a linear-elastic bay restoring force and story drift relationship as well as the slab for in-plane shear deformation. A constant $k_1$ means a stiffness ratio of bay 1, $K_1$, to a sum of two bays' stiffnesses $K_1$ and $K_2$. The ratio is the following

$$K_1 = 2K_2 \qquad \therefore k_1 = \frac{K_1}{K} = \frac{2}{3} \qquad \text{where } K = K_1 + K_2$$

This analytical model assumes the following:

a. A 8 degree-of-freedom system with 8 masses (see Figure 1. (b))
b. Uniform mass distribution on the slab which could be expressed by Table 1 and Figure 1, which is independent of mass on bay 1
c. Each mass with a same distance can move unidirectionally same as the seismicity.
d. Each bay is expressed as a mass supported by a shear spring which corresponds to its story drift.
e. Similarly the slab is expressed as 6 masses connected with 7 shear springs which correspond to its in-plane shear deformation. The springs have linear-elastic characteristics.
f. No flexural deformation of the slab is considered. Therefore the span L does not have anything to do with analytical results. This model does not incorporate the span L.

Newton's equation of motion for the system is

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{1\}\ddot{x}_G \tag{1}$$

where $\ddot{x}_G$ is ground acceleration, $\{\ddot{x}\}$ is acceleration vector, $\{\dot{x}\}$ is velocity vector, $\{x\}$ is motion vector, $[M]$ is mass matrix, $[C]$ is damping matrix, $[K]$is stiffness matrix

$$\{\ddot{x}\} = \{\ddot{x}_1 \quad \ddot{x}_2 \quad \cdots \quad \ddot{x}_{n-1} \quad \ddot{x}_n\}^T$$

$$\{\dot{x}\} = \{\dot{x}_1 \quad \dot{x}_2 \quad \cdots \quad \dot{x}_{n-1} \quad \dot{x}_n\}^T$$

$$\{x\} = \{x_1 \quad x_2 \quad \cdots \quad x_{n-1} \quad x_n\}^T$$

$n$ : number of mass

$$[M]=\begin{bmatrix} m'_1 & & & & & & \\ & m'_2 & & & & & \\ & & \cdot & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & m'_{n-1} & \\ & & & & & & m'_n \end{bmatrix}$$

$m'_i$ : distributed mass ($i$ = 1 ~ $n$)

$$[K]=\begin{bmatrix} K_1+K'_f & -K'_f & & & & & \\ -K'_f & 2K'_f & -K'_f & & & & \\ & \cdot & \cdot & \cdot & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & -K'_f & 2K'_f & -K'_f \\ & & & & & -K'_f & K_2+K'_f \end{bmatrix}$$

$K_i$ : bay stiffness ($i$ = 1, 2)
$K'_f$ : slab stiffness

The variable structural characteristics in the analytical investigations include the following.

$k_f$ : Slab shear stiffness ratio defined as a stiffness ratio of the slab to the supporting bays

$$k_f = \frac{K_f}{K} = 0.1 \sim 1000 \qquad \text{where} \quad 7K_f = K'_f$$

$T_0$ : Natural period when the slab is a rigid diaphragm

$$T_0 = 0.33,\ 0.67,\ 1.00\ (sec)$$

$m_1$ : Ratio of mass considered to be supported by the bay 1 to a sum of masses of the system, to say 1.0

$$m_1 = \frac{M_1}{M} = 0.50,\ 0.67,\ 0.80$$

$$\text{where} \quad M_1 = \sum_{i=1}^{4} m'_i, \quad M = \sum_{i=1}^{8} m'_i$$

The distributed masses $m'_1 \sim m'_8$ are obtained as follows (See Table 1)

$$m'_1 = \frac{7m_1 - 3}{4}, \qquad m'_2 \sim m'_8 = \frac{1-m_1}{4}$$

A lumped mass system might be used as well as the uniformly distributed mass on the slab with a same distance.

## 3 EIGENVALUE ANALYSIS

Eigenvalues and eigenvectors are properties of the equations that simulate the dynamic behavior of a structure. Prior to time history analysis for ground motion of seismicity, those values are found to know how the behavior of undamped structures for free vibration is affected by the slab shear stiffness ratio $k_f$ defined above. Based on those values, a lot of emphasis should be on the effective modal mass ratio and the modal participation vector.

The effective modal mass ratio is defined as a ratio of the effective modal mass to the actual mass and used to judge how a vibration mode is significant. Modes with relatively high effective mass ratios can be readily excited by base excitation. A sum of all of the effective mass ratios must be 1.0. The variation of the effective modal mass ratio of 1st mode of vibration in case of the natural period with the rigid slab $T_0$ = 0.67 (*sec*) is shown in Figure 2 as a function of the slab stiffness ratio $k_f$. This figure indicates that the effective modal mass ratio for 1st mode with the slab stiffness ratio $k_f$ which is equal to or greater than 1.0 may be considered as 1.0. That means that the distributed mass system with the ratio $k_f$ in a range of 1.0 or greater could be transferred into a single-degree-of-freedom (SDOF) system in mode space. The modal participation vectors correspond to eigenvectors of several modes of vibration for a unit vector {1} shown in Eq. (1). Figure 3 shows the modal participation vector of 1st mode of vibration and the slab stiffness ratio $k_f$ relationships in case of $T_0$ = 0.67 (*sec*) with the mass ratio of the bay 1, $m_1$ = 0.50. It is apparent from observation that increasing the slab stiffness ratio $k_f$ in a range of 3.0 or greater may not cause variation of the participation vector of 1st mode. In other words, there is not a discrepancy between the different slab stiffness ratios $k_f$ correspond to the participation vector of 1st mode less than 5% at every single mass.

Although the analytical results in case of $T_0$ = 0.67 (*sec*) are illustrated here, in case of other two different values of $T_0$, to say 0.33 and 1.00 (*sec*), approximately same behaviors of 1st mode of vibration are found with regard to the effective modal mass ratio and the participation vector, respectively. Consequently the slab stiffness ratio $k_f$ of 1.0 is enough to find the general dynamic behaviors of the distributed mass systems by use of analytical results for 1st mode of vibration. However, in order to find the local deformation of the slab by using analyses for only 1st mode, the slab stiffness ratio $k_f$ equal to 3.0 or greater might be required.

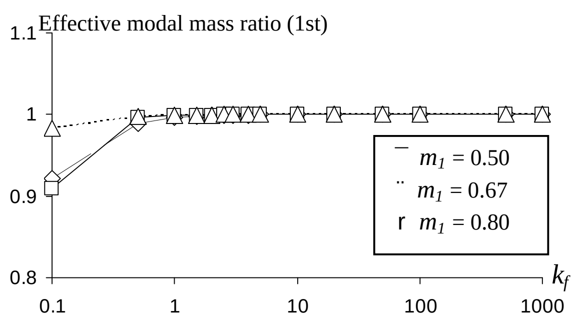



**Figure 2.** Effective modal mass ratio and slab stiffness ratio $k_f$ relationships ($T_0$ = 0.67)

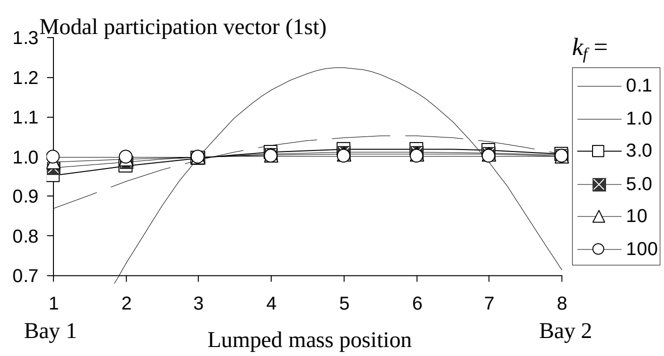



**Figure 3.** Modal participation vector and slab stiffness ratio $k_f$ relationships ($m_1$ = 0.50, $T_0$ = 0.67)

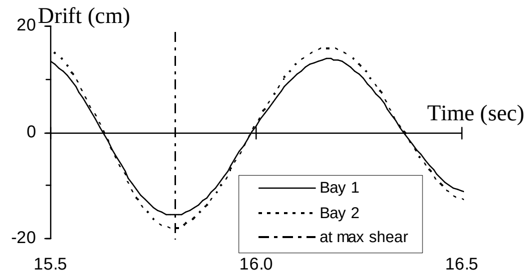



**Figure 4.** Story drift time histories at maximum slab shear for BCJ level-2 ($k_f$ = 1.0, $T_0$ = 0.67, $m_1$ = 0.5)

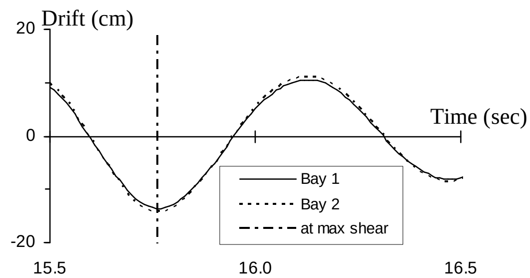



**Figure 5.** Story drift time histories at maximum slab shear for BCJ level-2 ($k_f$ = 3.0, $T_0$ = 0.67, $m_1$ = 0.5)

## 4 TIME HISTORY ANALYSIS

### 4.1 Seismic waves

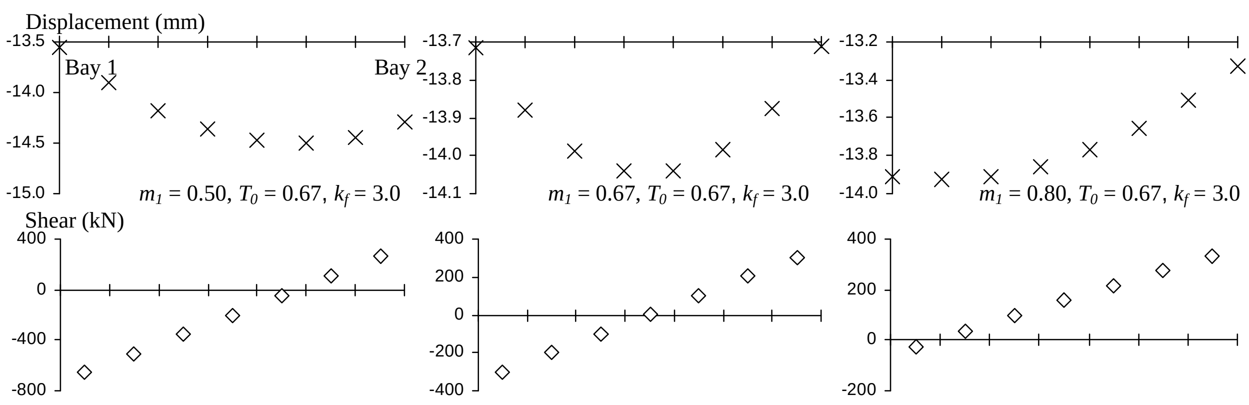
Applied unidirectional seismic waves are the Imperial Valley earthquake (EL Centro NS 1979) scaled such that its PGV (Peak Ground Velocity) matched 50 *cm*/*s* and the Building Center of Japan (BCJ 1994) level-2 [3], which is an artificial earthquake generated to be compatible with current Japanese seismic code with 50 *cm*/*s* of PGV. The integration is performed with a time step of 0.01 second during these respective seismic wave durations with two percent of critical damping to the initial stiffness of the bays.

### 4.2 Analytical results

Figures 4 and 5 indicate story drift time histories of the two bays for BCJ level-2 with two different values of $k_f$, 1.0 and 3.0, respectively. These figures indicates that whenever the maximum local shear response is generated in linear-elastic structures with slab stiffness ratio $k_f$ equal to or greater than 1.0, the both bays' story drift displacements are always their respective approximate peak values in the cycle. Both bays showed probably same time histories in a range of $k_f$ equal to 3.0 or greater (figure 5). The finding coincides with the behavior of the participation vector of 1st mode as mentioned above. So the slab diaphragm can be considered to be rigid in its own plane. Although the analytical results in case of $T_0$ = 0.67 are illustrated above, the entire results including three different values of $T_0$, to say 0.33, 0.67 and 1.00, have found that the natural period hardly affects the earthquake response in the slab shear behaviors.

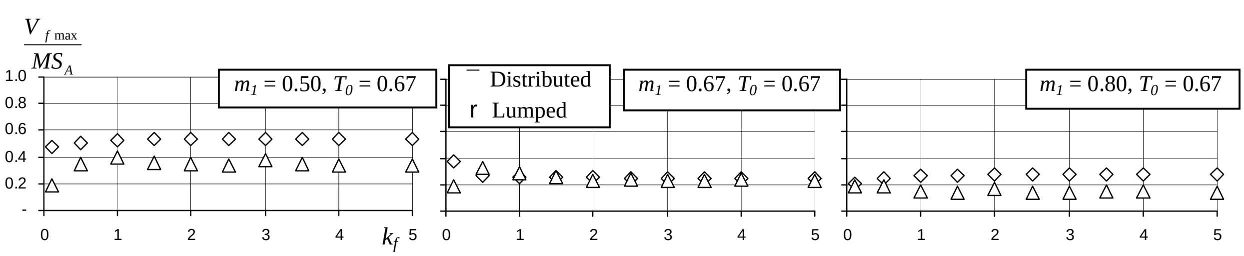
### 4.3 Differences of the slab shear response between lumped and distributed mass systems

Figure 6 displays mass displacement and local shear distributions in the slab for distributed mass when maximum local shear response occurred. In the figure, the displacement distributions are not linear. And local shear response variations along the axis normal to the direction of seismic motion can be seen. The shear distributions are not uniform and are approximately proportional to distance from the bays. These observations provide that inertial force applied to the mass on the slab could result in these behaviors. Maximum local shear response could be observed at either shear spring adjacent to the bays due to the inertial force for all the analytical cases including two different seismic waves and three different natural periods on the assumption of the rigid slab. Figure 6(b) shows an analytical case that the mass ratio $m_1$ (=0.67), the mass ratio of the bay 1 to a sum of all masses, equals the constant $k_1$ (=0.67), a stiffness ratio of the bay 1 to a sum of two bays. In this case time histories of displacements of bays 1 and 2 are approximately same as well as their fundamental natural periods, whereas local shear responses were observed. Then the average shear response was approximately zero. Moreover the local shear response in the middle of the slab is zero. From these facts the inertial force of mass could be thought transferred to the nearer bay. Meanwhile as for the other mass ratios of $m_1$ of 0.50 and 0.80, the shear distribution can be thought as a sum of the uniform shear due to difference of bay displacements and the proportional one as seen in cases of $m_1$ = 0.67. These findings could not be obtained by the lumped mass system. Although in the analyses a sum of the lumped masses equals that of the distributed masses, Nakamura et al. (2006, 2007) illustrated that the lumped mass matrix and consistent mass matrix (Archer 1963) for distributed mass in the lumped mass system resulted in probably same maximum slab shear response. Based on the fact, shear response comparisons between the lumped and distributed mass systems are conducted. Needless to say, the maximum dynamic local shear response intensity for the distributed mass system must be equal to or greater than the average value. Note that the major premise of the lumped mass system is uniform shear response in the slab. Figure 7 shows comparisons of the maximum local shear response between distributed and lumped mass systems for BCJ level-2. In this figure, $V_{f\,max}$ and $MS_A$ designate maximum local shear response and a sum of story shear response of the bays, respectively. As for the distributed mass system, inertial force applied to the mass on the slab could result in additional in-plane shear force of the slab. It is apparent that the lumped mass system could underestimate the maximum local shear response in the slab of the distributed mass, where the maximum local shear means a ratio of the maximum shear to a sum of story shear. This suggests necessity of a new formula for predicting that.




(a) $m_1$ = 0.50 (b) $m_1$ = 0.67 (c) $m_1$ = 0.80

**Figure 6.** Distributions of mass displacement and local shear response for BCJ level-2




(a) $m_1$ = 0.50 (b) $m_1$ = 0.67 (c) $m_1$ = 0.80

**Figure 7.** Local shear response comparisons between distributed and lumped mass systems for BCJ level-2

## 5 PREDICTION OF SHEAR RESPONSE

### 5.1 Proposal of Predictable Formulae

Nakamura et al. (2006, 2007) reported predictable formulae of the shear response based on balance of static force for the lumped mass system without mass on the slab as follows. Eq. (2) assumes a slab with a certain stiffness not rigid, meanwhile Eq. (3) does a rigid one. In case of the rigid slab, every single mass displacement must be always equal as well as the absolute acceleration. Thus each inertial force of the mass is proportional to the mass. Moreover a story shear ratio of the bays is same as their elastic stiffness ratio. Eq. (3) means that a maximum of the sum of the story shear of the bays could attribute to the maximum shear response of the slab. When slab shear stiffness ratio $k_f$ is sufficient in Eq. (2), this formula could be approximately equal to Eq. (3). If the slab stiffness ratio $k_f$ is infinite, these formulae are identical.

$$V_{f\,\max} = K_f \left| x_1 - x_2 \right| = \frac{k_f \left| m_1 - k_1 \right|}{k_1 (1 - k_1) + k_f} MS_A \quad (2)$$

$$V_{f\,\max} = \left| m_1 - k_1 \right| (V_1 + V_2)_{\max} \quad (3)$$

where $x_1$ and $x_2$ designate story drifts of bays 1 and 2 (see Figure 1(b)), $V_1$ and $V_2$ designate each bay’s story shear response respectively. Then in the design procedure, $(V_1 + V_2)_{\max}$ is given as $MS_A$ ($S_A$: design spectral response acceleration).

When the mass ratio $m_1$ and the stiffness ratio $k_1$ of bay 1 are equal, both above formulae must yield zero slab shear response. On the other hand, these formulae could not take into account additional shear response due to inertial force of the distributed mass on the slab. Assume that inertial force of the mass might be transferred to the nearer bay based on the facts found in the previous section 4.3, the following Eq. (4) is proposed here. Considering practicality and convenience of the seismic design procedure, Eq. (5) is also proposed to predict the maximum local shear response, based on Eq. (3).

$$V_{f\,\max} = K_f \left| x_1 - x_2 \right| + \max\left( \sum_{i=2}^{n/2} m_i', \sum_{i=n/2+1}^{n-1} m_i' \right) S_A = \frac{k_f \left| m_1 - k_1 \right|}{k_1 (1 - k_1) + k_f} MS_A + \max\left( \sum_{i=2}^{n/2} m_i', \sum_{i=n/2+1}^{n-1} m_i' \right) S_A \quad (4)$$

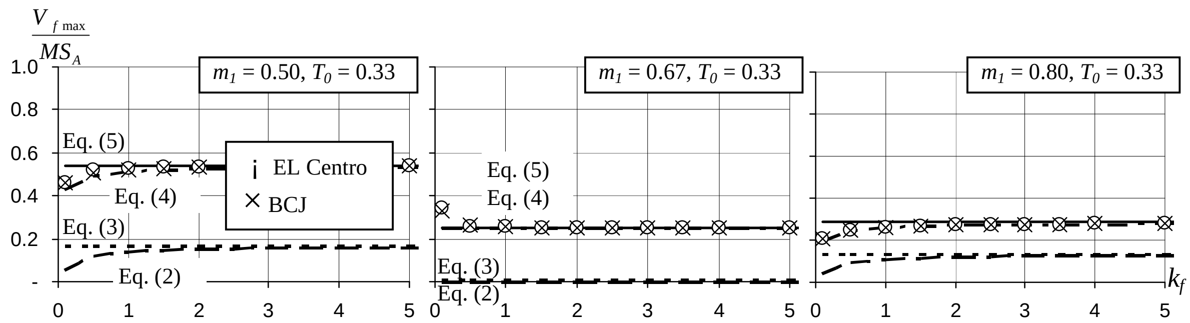
$$V_{f\,\max} = \left| m_1 - k_1 \right| MS_A + \max\left( \sum_{i=2}^{n/2} m_i', \sum_{i=n/2+1}^{n-1} m_i' \right) S_A \quad (5)$$

where $n$ : number of mass, $S_A$: response acceleration

### 5.2 Verification of Predictable Formulae for Slab Subjected to Shear Caused by Seismicity

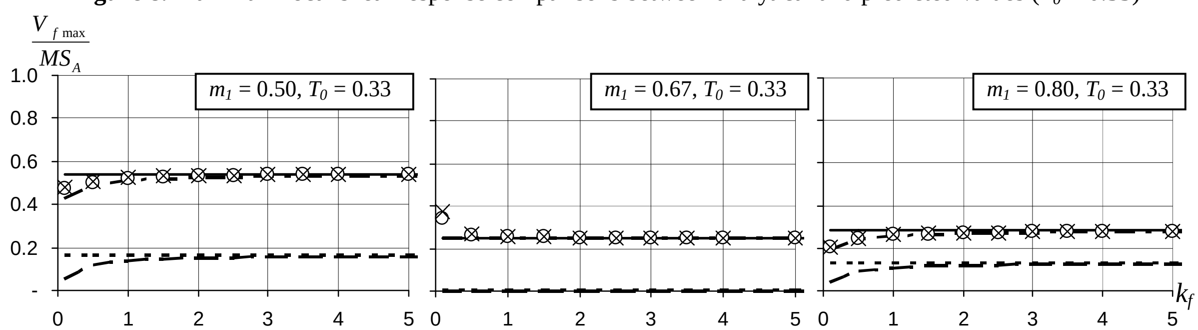
Figures 8 and 9 indicate maximum local shear response comparisons between analytical and predicted values with $T_0$ of 0.33 and 0.67 for the distributed mass system. In these figures ¡ and × symbols designate analytical results for EL Centro and BCJ respectively. When the slab stiffness ratio $k_f$ is equal to or greater than 3, increasing the value of $k_f$ could not influence the analytical slab shear response for the distributed mass system. The previously proposed Eqs. (2) and (3) could underestimate the maximum local shear response. Especially it is noted that in any cases of $m_1$ = 0.67 both previous formulae generate no shear response in-plane of the slab.

By taking account for inertial force applied to the mass on the slab, in addition to difference of the story drift between two bays supporting the slab, the maximum local shear response in the slab which stiffness




(a) $m_1$ = 0.50 (b) $m_1$ = 0.67 (c) $m_1$ = 0.80

**Figure 8.** Maximum local shear response comparisons between analytical and predicted values ($T_0$ = 0.33)




(a) $m_1$ = 0.50 (b) $m_1$ = 0.67 (c) $m_1$ = 0.80

**Figure 9.** Maximum local shear response comparisons between analytical and predicted values ($T_0$ = 0.67)

ratio $k_f$ in a range of 1.0 or greater could be predicted appropriately by Eq. (4). When the slab stiffness ratio $k_f$ is equal to or greater than 3.0, Eq. (5) is also available for the prediction as well as Eq. (4). In routine design procedures, Eq. (5) which assumes a slab to be rigid is preferable to Eq. (4) for its simplicity. That is to say, it is not required to know a difference of the displacement between the bays for calculation in Eq. (5).

## 6 CONCLUSIONS

In this study a variety of eigenvalue and time history analyses were conducted in order to investigate the dynamic local shear response behavior of the slab for the distributed mass system, which is single story with linear-elastic restoring force characteristics as well as the slab. The results and conclusions of the analytical studies presented in this paper may be summarized as follows:

a) Eigenvalue analysis showed that the distributed mass system with the slab stiffness ratio $k_f$ in a range of 1.0 or greater could be transferred into a SDOF system in mode space.
b) The slab stiffness ratio $k_f$ of 3.0 is enough to find the local deformation of the slab for the distributed mass system by only 1st mode of vibration.
c) Time History analysis showed that local shear response of the distributed mass system could differ from that of the lumped mass system.
d) The distributed mass system showed dynamic shear response variations along the axis normal to the direction of seismic motion, according to the mass inertial forces on the slab.
e) The maximum local shear response could be observed at either shear spring adjacent to the bays for all the analytical cases.
f) Inertial force of mass on the slab might be transferred to the nearer bay since in cases of approximately same displacements of supporting bays, the local shear responses in the middle of the slab were zero.

g) The lumped mass system might underestimate the maximum local shear response for the distributed mass system.
h) The previously proposed Eqs. (2) and (3) could underestimate the maximum local shear response of the distributed mass system as well.
i) The maximum local shear response in the slab which stiffness ratio $k_f$ in a range of 1.0 or greater could be predicted appropriately by newly proposed Eq. (4). When the slab stiffness ratio $k_f$ is equal to or greater than 3.0, Eq. (5) suitable for seismic design calculations is also available for the prediction.

Eigenvalue and time history analyses provided fundamental information described above upon the local shear response for the distributed mass system. The time history analyses suggest that mass distribution on the slab should be taken into account in the day to day seismic design of slab for in-plain shear. Two predictable formulae for maximum local shear response were proposed based on the analytical results. Comparisons between the formulae and the results provided verification and indicated their utility. This study focused on single-story linear-elastic systems. To provide comprehensive coverage on the seismic design requirements for slab shear response, multi-story and elasto-plastic systems must be investigated furthermore.

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